Abstracts

STOP 2011 Selected Topics of Operator Theory

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Summer School

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The Cosserat eigenvalue problem

The Cosserat eigenvalue problem is the Dirichlet problem for the Lamé equations of linear elasticity, where the Lamé parameter lambda (bulk modulus) is considered as the eigenvalue parameter. Estimates for the Cosserat eigenvalues are related to the Korn inequality and to the inf-sup constant for the divergence. The problem has a long history, starting with E. and F. Cosserat (1898), and contributions by Friedrichs (1937) and Mikhlin (1973). It has recently got more attention mainly from people in fluid dynamics, but also in electrodynamics and finite element analysis. Although there has been some progress, in particular for domains with corners, many very simple questions on this problem are still open, like the precise value of the lowest Cosserat eigenvalue for a square or a triangle.

In the talks, I will present the problem and its relations to some other problems of vector analysis and describe the classical results for smooth domains. Then I will show how Mellin analysis gives the essential spectrum for corner domains and present some computations for rectangles. Finally a relation with the spectral theory for a strongly singular volume integral operator in electromagnetism is analyzed.

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Calculus of partial differential operators on hypersurfaces and shell theory

Partial differential equations on Riemannian manifolds are usually written in intrinsic coordinates, involving metric tensor and Christoffel symbols. But we deal with a hypersurface, the Cartesian coordinates from the ambient space can be applied. The calculus is based on the Gunters derivatives, which are the projections of the classical coordinate derivatives from the ambient Euclidean space. Our scope in the present series of lectures is to deliver the recent results on the shell theory. It is well known that the theory of thin linearly elastic shells leads to two-dimensional equations on the middle hypersurface of the shell. We should apply the developed calculus of Gunter's derivatives and prove that when the thickness of the shell converges to 0, the equation of $3D$ elasticity converges exactly to the Lamé equation with constant Lamé coefficients on the middle surface. It is remarkable, that the applied asymptotic analysis allows to derive high order limit equations on the middle surface, which have the same simple form. The limit equation obtained by this process is much simpler than the one derived by Koiter-Sanchez-Palenzia-Ciarlet etc. with a similar asymptotic analysis, but based on the classical differential geometry.

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Advances in factorization theory

Factorization of matrix functions is a rich and still evolving theory, with applications to mathematical physics, integrable systems, random matrices, sampling and interpolation, to name a few. Two main problems in this theory are: (i) factorability criteria for matrix functions from various particular classes, and (ii) constructive and explicit factorization algorithms and formulas. In this series of lectures, we will attempt to systematize and report the developments in these two areas, with the emphasis on the progress in the last ten years. In particular, we will address the factorization problem in the ordered abelian group setting, allowing in particular to consider the classical Wiener-Hopf factorization and the almost periodic factorization from the unified point of view.

Workshop

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Canonical models and complex geometry

There are two distinct approaches to the study of contraction operators on a complex Hilbert space in the Cowen-Douglas class, one due to Sz.-Nagy and Foias involving harmonic analysis, and a second due to M. Cowen and the speaker involving complex geometry. Recently, Yun-Su Kim, Hyun Kwon, Jaydeb Sarkar and the speaker undertook a study to compare these two approaches using the language of Hilbert modules.

In this talk, I will describe the framework in which the Hilbert module defining the operator being studied is obtained as a quotient of standard Hilbert modules such as Hardy, Bergman or weighted Bergman modules on the unit disk. In the context of Sz.-Nagy and Foias, we consider canonical models for which the characteristic operator function is matrix valued having a regular inverse. One shows that the quotient module is in the Cowen-Douglas class and determine the associated vector bundle in terms of the multiplier. This enables one to characterize it in terms of the curvature of the Chern connection and its partial derivatives. These calculations hold not just for quotients built from the Hardy space, but also for weighted Bergman spaces and related spaces.

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Boundary integral equations recast as pseudodifferential equations

It is known that the treatment of boundary value problems based on variational principles often leads to corresponding boundary integral equations in weak formulations. Their mapping properties can then be derived from those of the associated partial differential equations. However, this approach is restricted only to those boundary value problems which can be formulated in terms of general variational principles based on Gärding's inequality. On the other hand, boundary integral equations can also be recast as special classes of pseudodifferential equations. In this lecture, we are concerned with the latter approach by applying pseudodifferential operator theory to a class of elliptic boundary value problems. In particular, the boundary value problems for the Helmholtz equation of scattering theory and the Lamé equations of linear elasticity will serve as model problems for illustrating the basic ideas how one can apply the theory of pseudodifferential operators and their calculus to obtain basic solution properties for the boundary integral equations.

This is a lecture on joint work with Wolfgang L. Wendland [Hsiao, G.C. and Wendland, W.L.: Boundary Integral Equations, Springer-Verlag, Berlin, Heidelberg 2008].

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The transient tunnel effect and Sommerfeld problem: Waves in semi-infinite structures

Our objective is the analysis of transient waves in two special semi-infinite structures: 1. the wave equation in two space dimensions with zero-Dirichlet-conditions imposed on a half-axis (Sommerfeld problem); 2. the Klein-Gordon equation on a star-shaped network composed of n half-axes connected at their origins, with potentials which are constant but different on each branch.

For 1 we prove the convergence of the resolvent of the spatial part of the problem in an appropriate topology, when approaching the spectrum (a Limiting Absorption Principle). For 2 we analyse the impact of tunnel effect on L-infinity-time-decay. The result is based on a generalized Fourier type inversion formula in terms of an expansion in generalized eigenfunctions and the Hörmander version of the stationary phase formula. The characteristics of the problem are marked by the non-manifold character of the domain. The more developped theory for the tunnel effect on a star-shaped network might serve as a guideline for further studies on transient Sommerfeld type problems.

These two problems can be viewed as examples for a general underlying strategy to analyze the behaviour of complicated compound systems by localization, which often leads to semi-infinite geometries. The price for the obvious advantage that different features of the structure can be isolated from one another, are various mathematical difficulties arising from the semi-infiniteness of domains or boundaries. The talk is based upon joint work with R. Haller-Dintelmann and V. Régnier.

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On Riesz minimal energy problems
(This is a lecture on joint work with H. Harbrecht, G. Of and N. Zorii)

In the n-dimensional Euclidean space, we study the constructive and numerical solution of minimizing the energy relative to the Riesz kernel of order $r - n$, where $1 < r < n$, for the Gauss variational problem, considered for finitely many compact, mutually disjoint, boundaryless $(n-1)$-dimensional Lipschitz manifolds being charged with Borel measures with prescribed sign. We show that the Gauss variational problem over an affine cone of Borel measures can alternatively be formulated as a minimum problem over an affine cone of surface distributions belonging to the Sobolev-Slobodetski space of order $(1-r)/2$ on the boundary. This allows the application of simple layer boundary integral operators and, hence, a penalty approximation. A corresponding numerical method is based on the Galerkin-Bubnov discretization with piecewise constant boundary elements. For $n = 3$ and $r = 2$, multipole approximation and in the case $1 < r < 3 = n$ wavelet matrix compression is applied to sparsify the system matrix. To the discretized problem, a projected gradient method is applied. Numerical results are presented to illustrate the approach.

References

[1] G. Of, W.L. Wendland and N. Zorii, *On the numerical solution of minimal energy problems*, Complex Variables and Elliptic Equations, 55 (2010) 991-1012.

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Contributed talks

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On some properties of Toeplitz operators and Riemann-Hilbert problems

A Riemann-Hilbert approach is applied to compare the dimensions of the kernels of two Toeplitz operators with related (matrix) symbols, using simple linear algebraic and complex analytic arguments. The results are applied to obtain some Fredholm type properties for operators with 2×2 symbols whose determinants admit a bounded factorisation.

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An index formula for classes of pseudodifferential operators

We consider here the problem of generalizing the Atiyah-Singer index theorem to operators on non-compact spaces, specifically to some classes of matrix valued pseudodifferential operators on $\mathbb{R}^n$. We first extend Atiyah and Singer's approach to operators with matrix symbols that are asymptotically constant, and obtain an index formula. We then apply this result to classes of pseudodifferential operators whose behavior at infinity is controlled by an extension to a suitable compactification, together with a given Lie algebra of differential operators that vanish at at infinity. Such a structure is called a Lie structure at infinity and defines, more generally, the so-called Lie manifolds. As particular cases, one obtains several well-known pseudodifferential calculi. We give Fredholm conditions depending on ellipticity and invertibility of a complete symbol and extend Atiyah-Singer's index formula to this class.

Our proof is mainly topological, in that we use homotopy of symbols and Bott periodicity. The results obtained can be applied to compute the index of perturbed Dirac operators with unbounded potentials.

This talk is based on joint work with V. Nistor.

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Mathematica software system and the study of some topics on operator theory

In this talk we show how the computer algebra system Mathematica allows us to study and to develop some topics on Operator Theory. In particular, we present some calculation techniques for computing singular integrals and algorithms for solving integral equations. The design of the algorithms is focused on the possibility of implementing on a computer all, or a significant part, of the symbolic and numeric calculations. We present several interesting examples.

This presentation is based on a joint work with José C. Pereira and Viktor G. Kravchenko.

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A functional analytic approach for a singularly perturbed non-linear traction problem in linearized elastostatics

We consider an application of an approach based on potential theory and functional analysis to analyze a non-linear traction problem of linearized elasticity in a domain with a small hole. The results are obtained in collaboration with Prof. Lanza de Cristoforis (Padova).

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Quantum wave guides with corners

Quantum wave guides are unbounded thin structures at the mesoscopic scale. The most widely considered have the shape of a wire which can be curved or twisted. The equation modelling the state of electrons in such structure is the plain Laplace equation with Dirichlet boundary conditions. The question of interest is the presence or absence of bound states, i.e. of eigenvalues of the Laplace-Dirichlet problem under the essential spectrum.

It has been proved that a plane wave guide with smooth boundary which

1. has a constant thickness,
2. is straight at infinity,
3. has a non-zero curvature somewhere,

has bound states under its essential spectrum.

Here we investigate the case of wave guides of V-shape, with one corner and constant thickness outside the corner. We show why it has always bound states as soon as it has a corner of opening $\theta < \pi$. We also describe the behavior of eigenpairs as the angle θ tends to zero: In this case, the number of eigenvalues under the essential spectrum tends to infinity and the eigenvectors have a multi-scale expansion. Their behavior at the corner in function of θ will be described.

These results are part of a joint work with Nicolas Raymond, and the numerical computations are done using the finite element library Mélina developed in Rennes.

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Compact perturbations of linear operators and its spectral properties

For an infinite dimension Banach space X, with $B(X)$ we denote the algebra of all linear bounded operators on X and $K(X)$ the ideal of all compact operators. For $T \in B(X)$, let $\sigma(T)$ be the spectrum of T, $\sigma_p(T)$, the set of all eigenvalues of T, and $\pi_0(T)$ the set of all isolated eigenvalues of finite geometric multiplicity.

The perturbation of an operator by some compact operators is a usual technic in areas of operators equations. Our interest is finding such compact operators that preserve some spectral properties of the original operator. In this talk we will give conditions that Weyl's type theorems and the continuity of the spectrum move from T to $T + K$, $T \in B(X)$ and $K \in K(X)$.

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More result on Toeplitz plus Hankel operators

In recent years, invertibility of Toeplitz + Hankel operators has been studied quite extensively. Explicit criteria have been obtain for operators of the form $T(a) + H(a)$ where a is piecewise continuous. In this talk I will present result about more general operators, $T(a) + H(b)$, where a, b are related by some condition. In these cases explicit invertibility criteria can be obtained in the case of piecewise continuous functions. In fact, the defect numbers can be evaluated. The setting is that of Hardy spaces $H^p(\mathbf{T})$. The talk is based on joint work with E.L. Basor.

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Localization operators on polyanalytic Fock spaces

In this talk we will explore the interplay between localization operators on the phase space and the structure of the Berezin symbol calculus in the context of polyanalytic Fock spaces.

Although localization operators has a long history of physics, they only became popular after the framework of Berezin (1988). From the mathematical point of view, localization operators were used at a first time by Daubechies (1988) in order to localize a signal simultaneously on time and on frequency. In the case that the symbol (window) is a Gaussian, the localization operators adopted by Daubechies are anti-Wick operators derived from Berezin quantization rules.

The main interest around this subject is the usefulness of these operators on the study of modulation spaces by means of methods from analytic function spaces like BMO^p spaces (cf. Coburn, Isralowitz & Li, 2011) combined with the Gelfand-Shilov framework (cf. Gelfand and Shilov, 1968). Beyond this study is the Coburn conjecture.

This conjecture announced by L. A. Coburn (2001) asserts that any Gabor-Daubechies operator with window ψ and symbol $a(x,\omega)$ quantised on the phase space by a Berezin-Toeplitz operator with window Ψ and symbol $\sigma(z,\overline{z})$ coincides with the Toeplitz operator with symbol $D\sigma(z,\overline{z})$ for some polynomial differential operator D. Recently the proof of this results was obtained by Lo (2007) and Englis (2009) in the context of Segal-Bargmann-Fock spaces for nicer class of windows under the constraint that the σ belongs to $L^{\infty}(\mathbb{C})$.

On the sequel we will present an extension of this result to polyanalytic Fock spaces using the Gabor analysis framework. Some examples and further applications of this result will be discussed in the context of *multiplexing* of signals.

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Connectedness of spectra of Toeplitz operators on Hardy spaces with Muckenhoupt weights over Carleson curves

Harold Widom proved in 1966 that the spectrum of a Toeplitz operator $T(a)$ acting on the Hardy space $H^p(\mathbb{T})$ over the unit circle $\mathbb{T}$ is a connected subset of the complex plane for every bounded measurable symbol a and $1 < p < \infty$. In 1972, Ronald Douglas established the connectedness of the essential spectrum of $T(a)$ on $H^2(\mathbb{T})$. We show that, as was suspected, these results remain valid in the setting of Hardy spaces $H^p(\Gamma, w)$, $1 < p < \infty$, with general Muckenhoupt weights w over arbitrary Carleson curves Γ.

This is a joint work with Ilya Spitkovsky.

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Nonlocal C^*-algebras of two-dimensional singular integral operators

Applying a local-trajectory method elaborated for studying nonlocal C^*-algebras associated with C^*-dynamical systems and the description of C^*-algebras generated by isometries, we construct a Fredholm symbol calculus for the C^*-algebra $\mathfrak{B}$ generated by the C^*-algebra $\mathfrak{A}$ of two-dimensional singular integral operators with continuous coefficients on a bounded closed simply connected domain $\overline{U} \subset \mathbb{R}^2$ with Liapunov boundary and by all unitary shift operators $W_g : f \mapsto J_g^{1/2}(f \circ g)$ where g runs a discrete amenable group G of quasiconformal diffeomorphisms of $\overline{U}$ onto itself with Hölder partial derivatives and the Jacobian J_g, and G acts on $\overline{U}$ topologically freely. As a result we establish Fredholm criteria for the operators $A \in \mathfrak{A}$ and $B \in \mathfrak{B}$.

The talk is based on a joint work with V. A. Mozel.

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On the kernel of some singular integral operators with shift

We consider the operator $T = I - cUP_+ \colon L_2^n(\mathbb{R}) \to L_2^n(\mathbb{R})$, on the real line, where I is the identity operator, $c \in C^{n\times n}(\overset{\circ}{\mathbb{R}})$ is a continuous matrix function on $\overset{\circ}{\mathbb{R}} = \mathbb{R}\cup\{\infty\}$, the one point compactification of $\mathbb{R}$, $(U\varphi)(t) = \varphi(t+\mu)$, $\mu \in \mathbb{R}$, is the shift operator, and $P_\pm = \frac{1}{2}(I \pm S)$ are the complementary projection operators, with $(S\varphi)(t) = (\pi i)^{-1} \int_{\mathbb{R}} \varphi(\tau)(\tau - t)^{-1} d\tau$ the operator of singular integration with Cauchy kernel. It is supposed that all the eigenvalues of the matrix $c(t)$ at ∞, simultaneously belong either to the interior of the unit circle $\mathbb{T}$ or to its exterior. Under these conditions, estimates for the dimension of the kernel of the operator T are obtained. We obtain analogous estimates, under similar conditions, for an operator with polynomial coefficient relative to the shift operator.

This is a joint work with Viktor G. Kravchenko.

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A singularly perturbed nonlinear Robin problem in a periodically perforated domain

This talk is devoted to the analysis of a singularly perturbed nonlinear Robin problem for the Laplace operator in a periodically perforated domain. We consider a sufficiently regular bounded open connected subset Ω of $\mathbb{R}^n$ such that $0 \in \Omega$ and such that the complement of the closure of Ω is connected. Then we choose a point $p \in]0,1[^n$. If ϵ is a small positive real number, we introduce the periodically perforated domain $S(\epsilon)^-$ obtained by removing from $\mathbb{R}^n$ the closure of the set $\cup_{z\in\mathbb{Z}^n}(p + \epsilon\Omega + z)$. For each positive and small ϵ, we consider a particular nonlinear Robin problem for the Laplace operator

in the set $S(\epsilon)^-$. More precisely, we consider a nonlinear Robin condition on the boundary of the set $p + \epsilon\Omega$, together with a periodicity condition. Under suitable assumptions, we show that for $\tilde{\epsilon} > 0$ sufficiently small, such a boundary value problem admits a locally unique family of solutions $\{u(\epsilon, \cdot)\}_{\epsilon \in]0,\tilde{\epsilon}[}$. Then we show that (suitable restrictions of) $u(\epsilon, \cdot)$ can be continued real analytically in the parameter ϵ around $\epsilon = 0$. Such an approach is alternative to those of asymptotic analysis and of calculus of variations.

The talk is based on joint work with Massimo Lanza de Cristoforis, University of Padova.

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On some boundary value problems for the Helmholtz equation in a cone of 240^o

We will present the explicit solution in closed analytic form of Dirichlet and Neumann problems for the Helmholtz equation in the non-convex and non-rectangular cone $\Omega_{0,\alpha}$ with $\alpha = 4\pi/3$. Actually, these problems are the only known cases of exterior (i.e., $\alpha > \pi$) wedge diffraction problems explicitly solvable in closed analytic form with the present method. To accomplish that, we reduce the BVPs in $\Omega_{0,\alpha}$ each to a pair of BVPs with symmetry in the same cone and each BVP with symmetry to a pair of semi-homogeneous BVPs in the convex half cones. Since $\alpha/2$ is an (odd) integer part of 2π, we obtain the explicit solution of the semi-homogeneous BVPs for half cones by so-called Sommerfeld potentials (resulting from special Sommerfeld problems which are explicitly solvable).

The talk is based upon joint work with Frank Olme-Speck.

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Nest algebras, invariant subspaces and ideals

Associative ideals of nest algebras are well understood, a complete characterisation existing for those that are weakly closed. Building on this knowledge, some progress has been made in finding a description for the ideals arising from other products on $B(H)$ such as the Jordan and the Lie products. In this talk we give a brief overview of the ideal structure of nest algebras and present some recent developments obtained for Lie ideals of continuous nest algebras.

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Hele-Shaw flow with a small obstacle

The joint work with Prof. G. Mishuris (Aberystwyth University, UK) is presented.

Asymptotic analysis of the flow passing a small obstacle in the Hele-Shaw cell is performed.

First we describe the geometry corresponding to the considered situation and present the (real-variable) Hele-Shaw model in a domain with an obstacle. This model is reduced to the form containing an unknown parametrization of the fluid domain and an unknown Green's (Robin) function of the mixed boundary for the Laplace operator related to this geometry.

The work is devoted to the asymptotic study of the solution to the Hele-Shaw boundary value problem in a domain with an obstacle. The results are based on the asymptotic formulas for Green's and Robin functions recently

obtained by V. Maz'ya and A. Movchan. The diameter of an obstacle plays the role of the small parameter here. Numerical calculation and geometric description of a different scenario of the flow's behaviour, related to different initial configuration of the flow region and an obstacle, is presented. The discovered features of the flow, as well as possible further study are discussed.

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Operators, symbols, and factorization

The factorization idea for solving different types of equations has a long history. Here we are speaking on a factorization in Wiener-Hopf sense, which permitted to describe the full solvability situations for the classical Riemann problem. But themultivariable analogue of such a factorization (wave factorization) is not so effective although for certain situations it permits to obtain interesting conclusions. Some problems are related to the solvability of pseudo differential equations in canonical non-smooth domains, which can be studied with the help of such a factorization, are described.

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The Picard operator

In this talk, we look at two partial differential operators related to electromagnetic scattering problems. The first is the Maxwell operator

$$\mathrm{M} = \mathbf{curl}\,\mathbf{curl} - k^2\,,$$

and the second is the Picard operator

$$\mathrm{P} = \begin{pmatrix} \mathbf{curl}\,\mathbf{curl} - k^2 & -\nabla \\ -\mathbf{div} & 1 \end{pmatrix}.$$

The operator M has been intensively studied in the context of boundary element methods (BEM). It is proved in [1, 2], for instance, that M leads to uniquely solvable variational formulations and that the Galerkin solution shows quasi-optimal convergence. The BEM derived from M are called the classical BEM. It is well known that the classical BEM are not stable when passing to the limit $k \to 0$. In the first part of this talk, we explain the reason why the limiting case causes problems. Motivated by [3], the Picard operator P has been constructed to resolve the low frequency problem. The second part of this talk is about P. Of special interest are the BEM derived from it. We call them the stabilized BEM. To show the unique solvability of the stabilized BEM, we use an ellipticity argument leading back to the classical BEM. To justify that P yields a stable numerical method for $k \to 0$, the algebraic properties of the discretization are discussed in the final part of the talk.

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